Stewart Shapiro. ***Thinking about mathematics. The philosophy of mathematics.*** Oxford University Press, Oxford and New York 2000, xiii + 308 pp.

Stewart Shapiro has written a very good, very useful textbook on the philosophy of mathematics. The book is also sorely needed, for while there are a few textbook-style overviews of the philosophy of mathematics in the literature, they are all badly out of date. What is good about Shapiro's book is that (i) a large chunk of the book (about a third) is dedicated to bringing the reader up to date on contemporary work in the philosophy of mathematics, and (ii) it takes the old logicism-formalism-intuitionism debate for what it was, namely, as a historical episode, and not as somehow capturing the three main views that one might endorse about mathematics. To most contemporary writers, all three of those views are mistaken (though, of course, there are still some contemporary advocates, and Shapiro covers them). It seems to me that there was something of a divide in the middle of 20th-century philosophy of mathematics: there were very few significant works published on the metaphysics and epistimology of mathematics between 1951, the year that Curry (certainly an old-schooler) published his book on formalism, and 1965, the year that Benacerraf published the first of the two famous papers that, in my view, ushered in the contemporary era in the philosophy of mathematics. The latter era is now a significant intellectual episode in its own right, and it is about time we had a book that provided a survey of this episode. Shapiro's book does this and does it well.

The book is divided into four parts. Part I provides some background for the rest of the book. Shapiro begins by discussing the question of whether the philosophy of mathematics ought to *dictate* how mathematics should be practiced or *account* for how it is practiced; he points out that the latter view is standard today. Then he discusses the notions of necessity and a priority, and describes the three main ontological views about mathematics (realism, idealism, and nominalism). Next, Shapiro discusses what he calls realism-in-truth-value, "the view that mathematical statements have objective truth-values, independent of the minds, languages, conventions, and so on of mathematicians" (p. 29). (Shapiro claims that some ontological anti-realists, e.g. Field and the intuitionists, are also anti-realists-in-truth-value, but that others, e.g. Chihara and Hellman, are realists-in-truth-value. One might question this characterization of Field: Shapiro points out that for Field, mathematical statements have only vacuous truth values; but Field thinks that, for example, '3 is prime' is objectively false, i.e., false independently of us, because of the nature of the world. It seems to me that there is a sense in which Field's view is more realist than Chihara's or Hellman's, semantically speaking, because unlike them, he is a *face-valuist* about the truth conditions of mathematical statements.) After this, Shapiro discusses the problem of the applicability of mathematics, and then he ends Part I by running through some of the particular mathematical propositions that philosophers have been especially interested in, for example the Löwenheim–Skolem theorem, the continuum hypothesis, and the incompleteness theorem.

Part II begins the historical survey of the philosophy of mathematics. In particular, it covers Plato, Aristotle, Kant, and Mill. The section on Kant is particularly good. Anyone who has ever tried to summarize Kant's philosophy of mathematics knows how difficult it can be to pin this view down. Shapiro does an excellent job of pulling the various strands together. He also does a good job making Mill's view seem more plausible than it is sometimes made to seem. For example, Shapiro points out that Mill has responses to several of the standard Fregean arguments against his view.

Part III is about logicism, formalism, and intuitionism. Within the logicist camp, Shapiro covers Frege, Russell, Carnap, and the contemporary neo-logicists, Wright and Hale. In connection with formalism, he begins with the views that Frege attacked, namely, those of Heine and Thomae, which, as Shapiro points out, both contain elements of term formalism and game formalism; Shapiro then discusses the early Hilbert's deductivism, the later Hilbert's finitism (and the Hilbert program), and Curry's metamathematical version of formalism. (In addition to these, I would have liked to see a discussion here of the early Putnam, who is a more contemporary advocate of deductivism.) As for intuitionism, Shapiro covers Brouwer, Heyting, and Dummett. Overall, Shapiro's treatment of all these philosophers is excellent.

(One thing I would have liked included in Part III is a discussion of the strangeness of the very fact that these three views were once thought of as opponents and, indeed, the only "live options" in the philosophy of mathematics. In fact, the three views here are all compatible with one another—they are not even answers to the same question—and as subsequent work has shown, they do not come close to exhausting all of the viable philosophies of mathematics. Indeed, it seems to me that formalism is the only one of them that even counts as a full-blown philosophy of mathematics, for it is the only one that provides a semantics and ontology for mathematics. Logicism and intuitionism were historically aligned with certain semantic and ontological views, but this is not central to those views—the one is a thesis about the logical relationship between mathematics and logic, and the other is a thesis about the cogency of certain kinds of mathematical proofs—and what is more, it seems to me that both of them can, with just as much plausibility, be aligned with radically different semantic and ontological views.)

Finally, Part IV covers contemporary views. It focuses, as it should, on views about the semantics and ontology of mathematics, in particular, on the question of whether or not mathematical realism is true. On the realist side of the debate, Shapiro discusses the views of Gödel, Quine, and the early Maddy. On the anti-realist side, he covers the views of Field, Chihara, Burgess and Rosen, Azzouni, and myself. He then ends with a chapter on structuralism, covering the views of Benacerraf, Hellman, Resnik, and himself. I am not so sure that structuralism really deserves to be singled out in this way; if I were organizing a book like this, I would include Resnik–Shapiro *ante rem* structuralism with the other contemporary realist views, and I would include Hellman's modal structuralism with the other contemporary anti-realist views. But of course, this is Shapiro's book, not mine, and it is understandable that he thinks that structuralism deserves a bit more space.

For the most part, Shapiro's discussions of the various contemporary views are accurate and very good. If I have any complaint at all here, it is that I think he might have devoted a bit more space, in the chapter on realism, to bringing out the different varieties of post-Gödel–Quine ontological realism that have emerged in recent years, and perhaps more importantly, to the different ways in which realists have tried to answer the Benacerrafian epistemological challenge. Shapiro says quite a bit on this topic in various places in the book—for example, in the chapter on structuralism, he discusses the ontology and epistemology of (his and Resnik's) *ante rem* structuralism, and in the chapter on anti-realism, he describes my own work on the ontology and epistemology of realism—but I think it would have been nice for the reader to see the various contemporary realist views brought together.

In sum, Shapiro has written an excellent book. It will be most useful, I think, as a secondary source for (graduate and undergraduate) students in philosophy of mathematics courses; and for graduate students and professional philosophers who do not know much about the philosophy of mathematics but want to be introduced to the area (anyone with a half-way decent philosophical background should be able to follow the book); and for professional philosophers of mathematics, who can use the text as a quick (but, of course, incomplete) source book on the views of the various figures that Shapiro covers.

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